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## Are There Statistically Impenetrable Contexts?



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### ABSTRACT

Bayesians claim that reason is conducted according to the axioms of probability. In addition, to this, they claim that Bayesian methodology gives us an account of the scientific method—an account that describes the actual practice of science. These are two claims. Bayesian decision theory falls short of its descriptivist claims due to psychological, contextual, and qualitative shortcomings. In some contexts, there may be problems with prediction and prior probability. Even though there might be an acceptable conclusion to a problem, Bayes' cannot even describe it. I use the example of a disjunction, wherein we cannot assign prior probabilities to either disjunct due to qualitative differences. Bayes only deals with degrees of personal belief. Here we run into not only the problem of assigning prior probability but also problems of uncertain evidence, and principles of relational acceptance. Not the descriptive aspect in jeopardy, but also the Bayesian conceptualization of the scientific method.



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## I. INTRODUCTION

Given certain assumptions, scientists may be unable to set the parameters required to begin sufficient testing of a hypothesis. Bayesian philosophers argue they are at the forefront of a formal logic that will render new information and increase our capacity to discern new instances of genuine knowledge. Like Bacon, the claim will be that all of the knowledge we supposedly had gained from the use of deduction, is old knowledge. What they are doing is rearranging all of the older knowledge contained in the first two premises. This was the charge that Bacon brought against the medieval scholastic philosophers. He argued for his enumerative induction by claiming that the point of philosophy was to find new arts and sciences. (Bacon, 1620) The deductive syllogism was stagnant. Bayesians claim their inductive method is descriptive. Scientists during investigation begin with subjective prior probabilities, probing into nature, “conditionalizing” until they reach more accurate results. In contrast, we are finding with more high-profile philosophers, that deductive logic is not entirely dead.<sup>1</sup>Naturalistic deductive logic now has a keen following, with its more tentative axioms. For those of us with a fondness for classical Aristotelian logic, this may be a good thing.

Bayesians need to provide a better account epistemically. There may be contexts wherein deduction has the descriptive edge. A challenge to strict Bayesians and deductivists is that research methodologies in science rely upon statistical analysis. This should not be taken as a blanket statement that all inductive models of inference are valid, however. A scientist looks upon a body of data, performs calculations, and comes up with a test statistic to conduct further experimentation. What philosophers since the time of David Hume have noted is that inductive methods such as these do not live up to the strict binary demands of deductive logical validity. Deduction may have another advantage over statistical inductive methods, such as Bayes, in that there may be situations in which the set of all possible outcomes of an experiment may elude a purely quantitative analysis.

Bayesian decision theory is the branch that claims the theorem is psychologically descriptive. Its level of application is limited, however, due to possible contextual complexities that may arise and the variety of ways in which human beings reason. Scientific settings pose problems in addition to the lack of adequate psychological description. To address the latter problem first, I

will use the decision-making scenario of going to the cinema. The variable stands for the proposition that I will go to see Dr. Tchivago at the Lagoon Theatre,  $q$  that my father will.

If either of us uses psychosocial factors to determine personal probabilities why he or I should go to the Lagoon theatre in Uptown, I would think we would be hard-pressed to find a pure numerical value to assign. The disjunction ( $p \vee q$ ) is a formal sentence that he goes or I do. What kind of probable values can we attach  $p$  or  $q$ ? Are there past similar instances that should come to the fore in our personalist decision-making procedure, such as past behavior in choice-making or movie-going? How do we choose between these possible past items on which we could plausibly model our thinking? This is why I chose the term “statistical impenetrability”. There can’t be a plausibly rational antecedent assignment of any value of a prior probability in these cases due to the number of possibly relevant considerations. In all epistemic honesty, this would have been an outright guess.

In cases where there is no “sample space” is Bayes’ a valid “decision-making procedure”? Bayesians might call the antecedent to guess a “noninformative prior”, although this method of scientific reasoning has its critics, to be sure.<sup>2</sup> If Bayesians claim that probabilistic reasoning is descriptive of a psychological fact about ourselves, they need to give an account, here. If it falls short, then it does not seem rightly descriptive. What function is an inductive inference supposed to serve? The instrumentalist answer is they allow scientists to infer universal generalizations which allow prediction and control—arguably a cornerstone of scientific inquiry. If statistically impenetrable contexts exist, Bayesians don’t have a descriptive claim, since we need an antecedent decision-making procedure to discern how to appropriately begin assigning values. Firstly, it doesn’t seem like we just guess; secondly, Bayes’ cannot deal with singular instances wherein reiterative use is requisite for kinematic operation. Bayesians might have to give up the thesis that their account is descriptive, which is highly undesirable.

## II. The Test Statistic

Important to the function of laboratory science is a phase of theory choice called “hypothesis testing”. Howson and Urbach in their *Scientific Reasoning: The Bayesian Approach*, criticize classical statistical analysis on several fronts. One reason is its inability to cope with Humean

worries. However, what is important is the notion of a test statistic in actual practice. If a sample estimate is compatible with the null hypothesis one will be able to tell with the test statistic. Scientists can administer a chi-square test, Neyman-Pearson test, Z-test, a Fisher test, etcetera. In contrast with what they call the “frequentist” view, subjective Bayesians claim that reasoning, in general, is modeled after Bayes’ theorem, which includes the concept of conditionalization. Herein lies a tension at the heart of probabilistic views in the philosophy of science. Should scientists become Bayesians, and or use the classical frequentist view?

Howson and Urbach claim that the classical view, “procedures for evaluating statistical hypotheses, such as significance tests, point estimation, confidence intervals, and other such techniques, provide an utterly false basis for scientific inference” (Howson &Urbach, 2014) That reasoning follows the axioms of probability is central to the Bayesian descriptivist claim. But also that we ought to base our thinking on betting behavior. Henri Poincare wrote that “The physicist is in the same position as the gambler who reckons up chances. Every time that he reasons by induction, he more or less consciously requires the calculus of probabilities” (Poincare 1952, p.184) One point in favor of Bayes’ is that it acknowledges the problem of induction raised by Hume. Conditionalization involves “updating rules” which allow one to take new data into account. If a sample of 4 out of 100 frogs in a pond behind a processing plant in an urbanized area have genetic mutations, and there is a substantial increase in such cases, Bayesian analysis will allow for the statistic to reflect that. The law of large numbers suggests the kinematic toward a more reliable statistic the more times we test. The classical view will come up with a more reliable statistic, say .10 which they can test against a population.

### **III. An Example of Statistical Impenetrability**

The frequentist versus Bayesian debate rages to this day, and perhaps natural deduction is less troublesome in some contexts. Let us illustrate this with our lay example in which Bayesian decision theory quite simply doesn’t work. The possibility of a predictive aspect of making a conditionalized judgment about my going to the cinema is lacking. We will need to keep in mind that the probability of .5 is simply shorthand for the idea that there are two outcomes of a scenario that must be 0 or 1, which would make our operations closed under Boolean analysis. Things become more complicated when we realize that the probabilities unless dealing with an unorthodox version of Bayes’, will both be subjective.

We have yet to see a satisfactory assessment of “either I ( $p$ ) will go, or ( $q$ ) my father will go” ( $p \vee q$ ). What are the prior probabilities of my going to the show over my father’s going to the show? They will be mutually exclusive, of course, due to the fact there is only 1 ticket. The possibilities are as follows  $\{(p \& \sim q), (\sim p \& q)\}$  There may be an immediate response to set the problem up .5 either way, another way of formalizing our sample space would be:  $(p \leftrightarrow \sim q) \vee (q \leftrightarrow \sim p)$ , assuming the condition that one person must go. Let us say my father’s subjective probabilities are such that 80% to his 20% are in my favor. He is elderly, he doesn’t want to find a place to park, walk around at night after the show, and so on. As a wild guess, this is how he assesses the disjunction. The factors might be qualitatively relevant and not quantitatively so. “Psycho-social” factors might come into play. How much I am interested in Russian history, for example. Can we assign a numerical value here? Would such considerations be excluded by Bayesian analysis?

If all we know is  $(p \cup q) = 1$ , I might be able to guess as a subjective prior probability assignment of 70% over my father’s 30% (I want to go). I am assuming here a mixed syllogism of some type although a classical statistical approach may prescribe an ANOVA test that takes more than one possible outcome. Statistical impenetrability suggests we are incapable of epistemically reaching ballpark prior probabilities, however. While an outsider might assign .5 to both, my thinking is .7 to .3, in favor of my getting the ticket. Lay factors abound: I love the Lagoon, I haven’t seen the movie in its entirety. . . The further we press on these fronts the more questions confront us. Should some considerations be weighed more than others, and how do we assign values? The problem is how to quantify these kinds of considerations that will have bearing on the outcome. Otherwise, epistemically, Bayesian decision theory seems to fall flat.

Bayes seems rather clumsy around qualitative issues like these. Kaplan, who is critical of Bayesians points out, “the orthodox Bayesian is, in short, guilty of advocating false precision. (Kaplan 1983, p.569). We have seen that the use of Bayes as a predictive tool relies upon a method of conditionalization. We need posterior to discern whether or not the prior probabilities of 70% to 30% are ballpark values. Here we are not given the luxury of repetition. There is only 1 ticket for the one night. An apt comparison is with the oft-cited frequentist example, flipping a coin. “In the long run,” the ratio of heads to tails should approximate 1/2.

But notice contrast our experiment did not mention that there would be multiple viewings of Dr. Zhivago involving my father and me at the Lagoon theater. . We have only one instance.

We still have only a prior probability as a complete guess and no instances of conditionalization. Despite whether I go to the movie or not, with merely this singular instance, Bayesian decision theory doesn't seem like it can have much to say. Assignment of prior probabilities seems utterly arbitrary. There might be semi-rational or non-logical concerns, although they look to be nevertheless relevant. Also, there is an assumption that all decisions only involve probable numerical values. At the very least, our scenario clearly illustrates how psychosocial complexities might have a bearing on choice, and statistical inference fails to provide an account.

#### IV. The Prior Probability Debate

That there may be non-numerical concerns points out yet another problem of prior probability. There is indeterminacy in these cases trying to assign them, despite whatever options we choose in our set of possible outcomes. The classic response of Bayesian philosophers of science is to choose noninformative priors. The conditionalization process will bring us closer to the true value. The law of large numbers states that the average of the results obtained should approximate the population the more times we experiment.

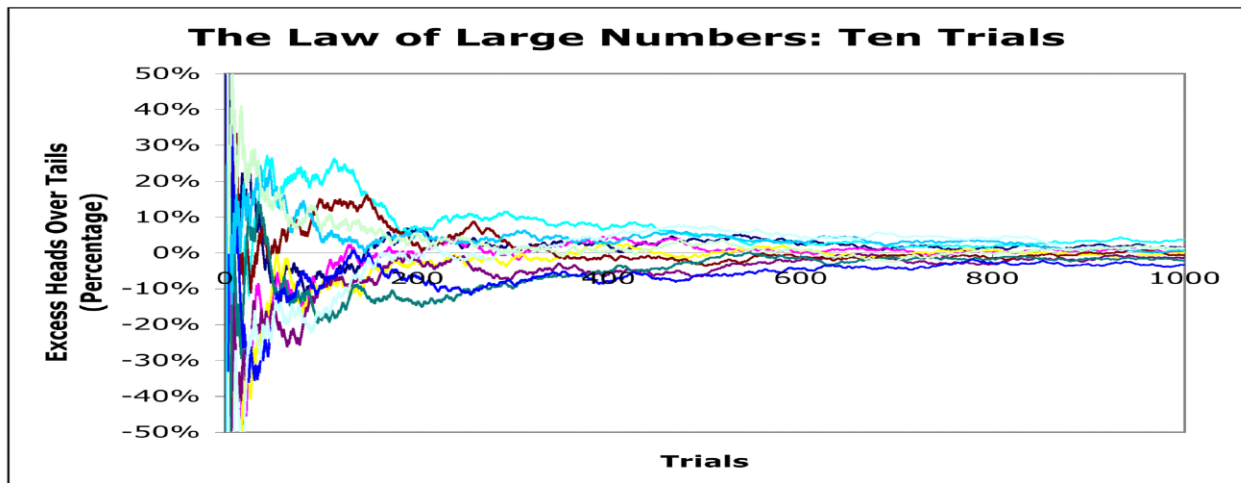


Fig. 1 Illustration of the law of convergence in respect to a number of coin tosses. David Foulke, "The Law of Large Numbers and Casino Earnings," *Alpha Architect*, 2014. <https://alphaarchitect.com/2014/01/the-law-of-large-numbers-and-casino-earnings/>.

Jacob Bernoulli wrote "even the stupidest man knows by some instinctive nature-that the greater the number of confirming observations, the surer the conjecture." (Sedlmeier, 1997, p.33) Along

with problems of prior probability come solutions. The retort more precisely is that they do not matter. If for a coin we think for some reason heads will come up .7, the law of convergence suggests that as time goes on, the chances of us coming up with a sound value of .5, increases, assuming it is a fair coin. Richard Jeffrey writes “your expectation of the relative frequency of success on the next  $m$  trials will equal the observed relative frequency  $s/n$  of success on the past trials.” (Jeffrey 1983, p.22)

Putatively, Bayes is saved by the nature of noninformative priors. These could be taken from a hat. It is not the case that they necessarily need to be based upon some putative source. “As new evidence accumulates, the probability on a proposition changes according to Bayes rule: the posterior probability of a hypothesis on the new evidence is equal to the prior probability of hypotheses on the evidence (Jeffrey 1983, p.133). Despite this, noteworthy critics such as Clark Glymour remain unpersuaded.

## V. A Scientific Scenario

Generally speaking, conditionalizing upon values with the use of Bayes’ theorem, and assigning prior probabilities, does not capture all of the relevant concerns. It would seem hard to assign a prior probability to the direction of an incalculably erratic atomic particle, for example. Take, for example, the dilaton, which was proposed by superstring theorists to explain why the universe formed as it did.

“Instead of our universe having constants like Newton’s constant or the Planck constant, the dilaton would have allowed these numbers to fluctuate during the early universe. After that, the dilaton would have frozen in value, which also caused the values of the fundamental constants to freeze. Dilatons might seem strange, but they are critical to understanding string theory cosmology. String theory relies on the Kaluza-Klein theories, and there is no way to ignore the dilaton in those theories. Physicists believe that the dilaton is a fundamental scalar in our universe, meaning that it is impossible to ignore it if it does exist.” (Brazier, 2015)

We know little about the dilaton although say we want to measure the direction it will take under certain conditions. The dilaton was proposed to explain why certain constants may fluctuate under certain circumstances, such as during the begging stages of our universe. It was thought

that the dilaton caused these fluctuations and then froze, and in our hypothetical scenario, we find it does not behave in a “frozen” manner at all. It behaves unlike any other particle we have ever seen. The conjunction of all the possibilities it could take in a Hadron collider experiment would be too monumental to calculate. Although the conjunction of all the projected directions it could take during our experiment may be transfinite due to it taking place in an enclosed space, we do not know the *nature* of this particular particle. There is nothing we can predict about it, although we know it is there due to its effects. There may be too many factors to take into consideration to begin the linear kinematic process. We need to know the relevant data set to take into account.

As Richard Feynman asks of particles in general: “how could we measure whether the particle is over here, or over there?” (mrtp, 2016) The probabilities here are that it might disappear, corrode, split in two, fracture into a million smaller particles, and anon. Our problem might be experimental design to set the parameters adequately. Some philosophers have argued that for a scientific setting to be legitimate, there has to be a set of possible answers in front of a scientist when he or she administers the test. But Bayesian kinematics, then, hardly seem to be an exhaustive description of how scientists reason. If they have learned anything from the Copenhagen interpretation of quantum mechanics, it should be to expect the unexpected.

## VI. CONCLUSION

Karl Popper once famously wrote “induction, i.e. inference based on many observations, is a myth. It is neither a psychological fact, nor a fact of ordinary life, nor one of the scientific procedures.” (Popper, 1963, p.63) Given the shortcomings of Bayesian decision theory as well as their descriptive claims, there may be new questions to ask. (a) Does natural deduction furnish us with the formal tools to describe scenarios Bayes cannot? If we shift now to deduction, (b) does Bacon’s admonishment that it gives us no new information hold? A Bayesian assumption is that if an event does not fit neatly within the Kolmogorov axioms, it simply will not work. That would, I think, include a large chunk of reality and our thinking about it. If I have not illustrated the point well, I hope I have at least directed some attention toward what I see as a considerable gap in what some consider to be a putative solution to the problem of induction. In the scenario with the 1 ticket, there is a standing disjunction ( $p \vee q$ ). William James believed that a proposition is meaningless until some action is taken upon it. (James, 1912) With either  $p$  or  $q$ , here, we *do*



have an answer. The Bayesian will be there trying to discern the proper data set and their prior probabilities. The linear kinematic process cannot begin.

Despite Howson and Urbach's potshots toward classical statistics, scientists use classical statistical analysis to this day. Bayes' is to be prescriptive, many of the insights in their book lose their bite. Indeed, one may accuse them of falling prey to the same fallaciousness they level at Popper and his hypothetico-deductivist method of inference.<sup>3</sup> Our recurrent question was whether or not Bayes was descriptive. We have a shortcoming here in predicting the behavior of the dilaton. In a case without past instances, we don't see to have a rational way to posit a prior. Larry Laudan comments in *Progress and its Problems* (1977) that providing an adequate model of rationality is the primary business of the philosopher of science but no extant methodologies fit actual science. Perhaps unless we either perfect abduction, or stay true to some form of deduction, this may remain the case.

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