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Determination of Temperature in an Elliptical Cylinder with Internal Heat Source

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Sunil D. Bagde*[1], Ujwala Beldar[2]

[1]Department of Mathematics, Gondwana University, Gadchiroli- 442605, India [2]Research Scholar, Department of Mathematics Gondwana University, Gadchiroli MIDC Road Complex, Gadchiroli-442605, India

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ABSTRACT

The present paper emphasizes the temperature distribution and unknown temperature in an elliptical cylinder utilizing internal heat which changes its place along the Z-axis. The Heat conduction in an elliptical cylinder with a moving heat source is analyzed. The temperature of the cylinder changes as the same limited area on the cylinder surface is heated by a heat source. The various boundary conditions of temperature have been evaluated by using finite Mathieu transform, finite Marchi -Fasulo transform and Laplace transform.

INTRODUCTION:

Heat Conduction problems are encountered in many engineering applications in various producing processes equivalent to attachment, metal cutting, drilling, grinding of metals and so many metals work. Due to temperature changes, material properties are likely to change.

This type of problem has the advantage of finding the temperature distribution at a prior state when the temperature distribution at any position is known at any instant. In recent years heat conduction problems for the circular boundary have been worked out by many authors.

Choubey [1] and Gupta [2] have defined the finite Mathieu transform and its properties. Masket [4] has applied Green functions. Mehta [3] has applied Marchi and Zgrablich transform. Cheng and Lin [8] have designed an analytical model which describes a three-dimensional temperature field in a finite thickness plate. Kidawa-Kukla [9] determined temperature distribution in a rectangular plate that was heated by a moving heat source. Gadle *et. al* [10] determined the temperature in a thin rectangular plate with an internal moving heat source. Sabherwal [7] studied inverse problem in heat conduction Marchi E and Zgrablich [5] has studies heat conduction in hollow cylinders with radiation. Bagde S.D. and Khobragade N.W. [11] study of heat conduction problem for a finite elliptical cylinder.

The present paper elaborates on the determination of temperature in an elliptical cylinder with an internal heat source we have applied finite Mathieu transform [1] and finite marchi-fasulo transform techniques to solve the problem with the boundary conditions.

STATEMENT OF PROBLEM

Heat flow in an elliptical cylinder of finite height with an internal heat source, we consider the heat conduction equation in an elliptic cylindrical shell $(a \le x \le b, 0 \le y \le 2\pi), \ -h \le z \le h$, when there are sources of heat within it that lead to the temperature distribution. If we assume that the rate of generation of heat is independent of the temperature and the length of the shell is finite, then the fundamental differential equation in elliptical coordinates is given by [6]

$$\frac{1}{\alpha}\frac{\partial F}{\partial t}=\frac{2d^{-2}}{(Cosh2x-Cos2y)}\left(\frac{\partial^2 F}{dx^2}+\frac{\partial^2 F}{dy^2}\right)+\frac{\partial^2 F}{dz^2}+\psi(x,y,z,t) \qquad (1)$$

Where α is constant known as diffusivity,

2d = interfocal length of ellipse and temperature distribution, T is the function of x, y and t the time function. $\psi(x, y, z, t)$ is taken for heat generation.

The boundary conditions are

$$F(a,y,z,t)+\beta\ 1\frac{\sqrt{2}}{y}(cosh2x-cos2y)^{-\frac{1}{2}}\frac{\partial F(a,y,z,t)}{\partial x}=M_a(y,z,t) \qquad (2)$$

$$t>0,\ 0\le y\le 2\pi,\ -h\le z\le h$$

$$F(b,y,z,t)+\beta_2\frac{\sqrt{2}}{y}(cosh2x-cos2y)^{-\frac{1}{2}}\frac{\partial F}{\partial x}(b,y,z,t)=M_b(y,z,t) \qquad (3)$$

$$t>0,\ 0\le y\le 2\pi,\ -h\le z\le h$$

Where β_1 , β_2 are radiation constants on the two surfaces.

$$F(x,y,z,t)=f(x,y,z) \text{ (Known) for all } a\le x\le b,\ 0\le y\le 2\pi \qquad (4)$$

$$F(x,y,z,o)=g(x,y,z) \text{ (unknown) for all } a\le x\le b\quad 0\le y\le 2\pi \qquad (5)$$

The equations (1) – (5) constitute the mathematical formulation of the problem under consideration.

SOLUTION OF THE PROBLEM

We applying finite Mathieu transform with respect to z define on [1] with region $a \le x \le b,\ \ 0 \le y \le 2\pi$ and satisfies the radiation type boundary conditions at the boundaries *a* and b then,

$$\overline{F}(q_{n,m}) = \int_a^b \int_0^{2\pi} (\cosh 2x - \cos 2y) K_{n,m}(\beta_1, \beta_2, x, y, q_{n,m}) dx.dy \qquad (6)$$

Where the kernel $K_{n,m}(\beta_1, \beta_2, x, y, q_{n,m})$ is given by

$$K_{n,m}(\beta_1, \beta_2, x, y, q_{n,m}) = Ce_n(x, q_{n,m}) ce_n(y, q_{n,m}).[Mey_n(\beta_1, a, y, q_{n,m}) + Mey_n(\beta_2, b, y, q_{n,m})]$$
$$- Mey_n(x, q_{n,m}) ce_n(y, q_{n,m}).Ce_n(\beta_1, a, y, q_{n,m}) + ce_n(\beta_2, b, y, q_{n,m})] \quad (7)$$

And $q_{n,m}$ is roots of

$$ce_n(\beta_1, a, y, q_{n,m}) Mey_n(\beta_2, b, y, q) - ce_n(\beta_2, b, y, q).Mey_n(\beta_1, a, y, q) = 0 \qquad (8)$$

Where $ce_n(y,q)$ *and* $Ce_n(x,q)$ are ordinary Mathieu function and modified mathieu function of first kind. $Mey_n(x,q)$ is modified mathieu function of the second kind.

Inversion of Finite Mathieu Transform

If $F(x, y)$ satisfies Dirichlet conditions in $a \le x \le b, 0 \le y \le 2\pi$ *and* $\overline{F}(q_{n,m})$ exit then,

$$F(x,y) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\overline{F}(q_{n,m}) K_{n,m}(\beta_1, \beta_2, x, y, q_{n,m})}{C_{n,m}} \qquad (9)$$

Where

$$C_{n,m} = \int_a^b \int_0^{2\pi} K^2_{n,m}(\beta_1, \beta_2, x, y, q_{n,m})(\cosh 2x - \cos 2y) dx.dy \qquad (10)$$

Summation being taken over the positive roots of (9).

By applying finite Mathieu transform defined in (1) and using the conditions (2), (3) and using the property of Mathieu transform given by Choubey [1], one obtains

$$\frac{d\overline{F}}{dt} - k\lambda_{2n,m}^{-2} = \chi(q_{n,m}t) \qquad (11)$$

Where

$$\chi(q_{n,m}t) = \overline{\psi}(q_{n,m}t) - \int_0^{2\pi} \begin{bmatrix} \frac{\alpha}{\beta_2} K_{n,m}(\beta_1, \beta_2, q_{n,m}b) M_b(y,t) \frac{h}{\sqrt{2}} (Cosh2b - Cos2y)^{1/2} \\ -\frac{\alpha}{\beta_1} K_{n,m}(\beta_1, \beta_2, q_{n,m}x) M_a(y,t) \frac{h}{\sqrt{2}} (Cosh2a - Cos2y)^{1/2} \end{bmatrix} dy \qquad (12)$$

and $\overline{\psi}(q_{n,m}t), \overline{F}(q_{n,m}t)$ is finite Mathieu Transform of $\psi(x,y,t)$ *and* $F(x,y,t)$ defined as follows:

$$\overline{\psi}(q_{n,m},z,t) = \int_0^{2\pi} \int_a^b \psi(x,y,z,t) K_{n,m}(\beta_1, \beta_2, q_{n,m}, x, y)(cosh2x - cos2y) dx\, dy$$

$$\overline{F}(q_{n,m},z,t) = \int_0^{2\pi} \int_a^b F(x,y,z,t) K_{n,m}(\beta_1, \beta_2, q_{n,m}, x, y)(cosh2x - cos2y) dx\, dy \qquad (13)$$

Therefore the solution of equation (11) is

$$\overline{F}(q_{n,m}, z, t) = e^{-\alpha\lambda_{n,m}^2 t} \int_0^1 \chi(q_{n,m}t^1) e^{\alpha\lambda_{n,m}^2 t} dt^1 + Ae^{-\alpha\lambda_{n,m}^2 t} \qquad (14)$$

Where A is an arbitrary constant, using the equations (1), (6), (10), and (14) we obtain

$$\overline{g}(q_{n,m}) = \overline{f}(q_{n,m}) e^{-\alpha\lambda_{n,m}^2 t} - \int_0^1 \chi(q_{n,m}t^1) e^{-\alpha\lambda_{n,m}^2 t} dt^1 \qquad (15)$$

Applying the inversion finite Mathieu transform we obtain

$$g(x,y,z,t) = \sum_{l=0}^{\infty} \frac{P_l(z)}{\lambda_l} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{1}{C_{n,m}} \left[\overline{f}(q_{n,m}) e^{\alpha\lambda_{n,m}^2 t} - \int_0^1 \chi(q_{n,m}t^1) e^{\alpha\lambda_{n,m}^2 t} dt^1 \right]$$

$$\times K_{n,m}(\beta_1, \beta_2, x, y) \quad (16)$$

CONCLUSION:

In this paper, we have investigated the temperature distribution in an elliptical cylinder with an internal heat source with the help of the Mathieu function and integral transform techniques. The results are obtained in the form of infinite series. The expressions that are obtained can be applied to the design of useful structures or machines in engineering application.

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 *Image Author -1*	***Author Name – Dr. Sunil D. Bagde*** *Assistant Professor,* *Gondwana University, Gadchiroli (M/S)* *India-442605*
 *Image* *Author -2*	***Ku. Ujwala Beldar*** *Research Scholar* *Gondwana University, Gadchiroli (M/S)* *India-442605*