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A Classification of Rectangles in Connection with Fascinating Number Patterns

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ABSTRACT

There are two sections I and II. **Section I** exhibits rectangles, where, in each rectangle, the area added with its semi-perimeter is represented by a special number. **Section II** presents rectangles, where, in each rectangle, the area minus its semi-perimeter is represented by a special number.

INTRODUCTION

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The diophantine problems connecting geometrical representations with special numbers, namely, Trimorphic numbers, Sphenic numbers, Harshad numbers, etc are presented in [1, 17]. This paper is devoted for obtaining rectangles with special characterizations in connection with fascinating numbers, namely, Woodall Numbers, Cullen Numbers, Motzkin Numbers.

It seems that the above problems have not been considered earlier.

Definitions:

Woodall Number:

A **Woodall number** (W_n) is any natural number of the form $W_n = n.2^n - 1$.

Cullen Number:

A **Cullen number** (C_n) is any natural number of the form $C_n = n.2^n + 1$

Motzkin Number:

Motzkin number (M_n) is the number of different ways of drawing non-intersecting chords between *n* points on a circle (not necessarily touching every point by a chord).

Method of Analysis:

Let R be a rectangle with dimensions x and y. Let A and S represent the Area and Semi-perimeter of R.

Section-I: $A + S =$ Woodall number

The problem under consideration is mathematically equivalent to solving the binary quadratic diophantine equation represented by:

$$xy + (x + y) = \alpha \qquad (I.1)$$

where α is a Woodall number.

Rewrite (I. 1) as

$$x = \frac{\alpha - y}{y+1} \tag{I.2}$$

Given α, it is possible to find x in integers for suitable y in integers. The following Table 1.1 exhibits the Woodall number with their corresponding rectangles satisfying (I.1):

Table No. 1.1: $A+S=\alpha$

Woodall number (α)	$R(x,y)$	Observations	
		Primitive rectangles	Non- Primitive rectangles
7	(1,3), (3,1)	2	-
23	(1,11), (2,7), (3,5), (5,3), (7,2),(11,1)	6	-
63	(1,31), (3,15), (15,3), (31,1)	2	2
159	(1,79), (3,39), (4,31),(7,19), (9,15),(15,9),(19,7),(31,4), (39,3),(79,1)	6	4
383	(1,191),(2,127),(3,95),(5,63), (7,47),(11,31),(15,23),(23,15), (31,11),(47,7),(63,5),(95,3), (127,2),(191,1)	14	-
895	(1,447),(3,223),(6,127),(7,111), (13,63),(15,55),(27,31),(31,27), (55,15),(63,13),(111,7),(127,6), (223,3),(447,1)	12	2
2047	(1,1023),(3,511),(7,255),(15,127), (31,63),(63,31),(127,15),(255,7), (511,3),(1023,1)	10	-
4607	(1,2303),(2,1535),(3,1151),(5,767), (7,575),(8,511),(11,383),(15,287), (17,255),(23,191),(31,143),(35,127), (47,95),(63,71), (71,63),(95,47), (127,35),(143,31),(191,23),(255,17), (287,15),(383,11),(511,8),(575,7), (767,5),(1151,3),(1535,2),(2303,1)	26	2
10239	(1,5119), (3,2559), (4,2047), (7,1279), (9,1023),(15,639),(19,511),(31,319), (39,255),(63,159),(79,127),(127,79), (159,63),(255,39),(319,31),(511,19), (639,15),(1023,9),(1279,7),(2047,4), (2559,3),(5119,1)	12	10

22527	(1,11263),(3,5631),(7,2815),(10,2047), (15,1407), (21,1023), (31,703),(43,511), (63,351),(87,255),(127,175),(175,127), (255,87),(351,63),(511,43),(703,31), (1023,21),(1407,15),(2047,10),(2815,7), (5631,3),(11263,1)	12	10
49151	(1,24575), (2,16383), (3,12287),(5,8191), (7,6143),(11,4095),(15,3071),(23,2047), (31,1535),(47,1023),(63,767),(95,511), (127,383),(191,255),(255,191)(383,127), (511,95),(767,63),(1023,47),(1535,31), (2047,23),(3071,15),(4095,11),(6143,7), (8191,5),(12287,3),(16383,2),(24575,1)	26	2
106495	(1,53247),(3,26623),(7,13311),(12,8191), (15,6655),(25,4095),(31,3327),(51,2047), (63,1663),(103,1023),(127,831), (207,511),(255,415),(415,255),(511,207), (831,127),(1023,103),(1663,63), (2047,51),(3327,31),(4095,25),(6655,15), (8191,12),(13311,7),(26623,3),(53247,1)	20	6

In a similar manner, Tables 1.2, 1.3 exhibit Cullen Numbers, Motzkin Numbers, along with their corresponding rectangles satisfying (I.1).

Table No. 1.2: $A+S=\beta$

Cullen number (β)	$R(x,y)$	Observations	
		Primitive rectangles	Non- Primitive rectangles
9	(1,4),(4,1)	2	-
25	(1,12), (12,1)	2	-
65	(1,32), (2,21), (5,10), (10,5), (21,2), (32,1)	4	2
161	(1,80),(2,53),(5,26),(8,17), (17,8),(26,5),(53,2),(80,1)	8	-
385	(1,192),(192,1)	2	-
897	(1,448),(448,1)	2	-
2049	(1,1024),(4,409),(9,204),(24,81), (40,49),(49,40),(81,24),(204,9), (409,4),(1024,1)	6	4
4609	(1,2304), (4,921), (9,460), (460,9), (921,4),(2304,1)	6	-
10241	(1,5120),(2,3413),(5,1706),(8,1137), (17,568), (568,17), (1137,8),(1706,5), (3413,2),(5120,1)	10	-

22529	(1,11264),(2,7509),(4,4505),(5,3754),(9,2252),(14,1501),(29,750),(750,29),(1501,14),(2252,9),(3754,5),(4505,4),(7509,2),(11264,1)	14	-
49153	(1,24576),(6,7021),(13,3510),(3510,13),(7021,6),(24576,1)	4	2
106497	(1,53248),(6,15213),(13,7606),(7606,13),(15213,6),(53248,1)	4	2

Table No. 1.3: $A+S=\gamma$

Motzkin number (γ)	$R(x,y)$	Observations	
		Primitive rectangles	Non- Primitive rectangles
9	(1,4),(4,1)	2	-
21	(1,10), (10,1)	2	-
51	(1,25), (3,12), (12,3), (25,1)	2	2
127	(1,63),(3,31),(7,15),(15,7),(31,3),(63,1)	6	-
323	(1,161),(2,107),(3,80),(5,53),(8,35),(11,26),(26,11),(35,8),(53,5),(80,3),(107,2),(161,1)	12	-
835	(1,417),(3,208),(10,75),(18,43),(21,37),(37,21),(43,18),(75,10),(208,3),(417,1)	8	2
2188	(10,198),(198,10)	-	2
5798	(2,1932),(1932,2)	-	2
15511	(1,7755),(3,3877),(6,2215),(7,1938),(13,1107), (27,553),(55,276),(276,55),(553,27),(1107,13),(1938,7),(2215,6),(3877,3),(7755,1)	14	-
41835	(1,20917),(3,10458),(10458,3),(20917,1)	2	2
113634	(4,22726),(22726,4)	-	2

Section- II: $A-S=$ Woodall number

The problem under consideration is mathematically equivalent to solving the binary quadratic diophantine equation represented by

$$xy-(x+y)=\alpha \tag{II.1}$$

where α is a woodall number.

Rewrite (II.1) as

$$x=\frac{\alpha+y}{y-1} \tag{II.2}$$

Given α, it is possible to find x in integers for suitable y in integers. The following Table 2.1 exhibits the Woodall number with their corresponding rectangles satisfying (II.1).

Table No. 2.1: $A-S=\alpha$

Woodall number (α)	$R(x,y)$	Observations	
		Primitive rectangles	Non- Primitive rectangles
1	(2,3), (3,2)	2	-
7	(2,9), (3,5), (5,3),(9,2)	4	-
23	(2,25), (3,13), (4,9), (5,7), (7,5), (9,4), (13,3), (25,2)	8	-
63	(2,65), (3,33), (5,17), (17,5), (33,3), (65,2)	4	2
159	(2,161), (3,81), (5,41), (6,33), (9,21), (11,17), (17,11), (21,9), (33,6), (41,5), (81,3), (161,2)	6	6
383	(2,385), (3,193), (4,129), (5,97), (7,65), (9,49), (13,33), (17,25), (25,17), (33,13), (49,9), (65,7), (97,5), (129,4), (193,3), (385,2)	16	-
895	(2,897), (3,449),(5,225),(8,129), (9,113),(15,65),(17,57),(29,33), (33,29),(57,17),(65,15),(113,9), (129,8),(225,5),(449,3),(897,2)	12	4
2047	(2,2049),(3,1025),(5,513),(9,257), (17,129),(33,65),(65,33),(129,17), (257,9),(513,5),(1025,3),(2049,2)	12	-
4607	(2,4609), (3,2305), (4,1537), (5,1153), (7,769), (9,577), (10,513), (13,385), (17,289), (19,257), (25,193), (33,145),	28	2

	(37,129), (49,97), (65,73), (73,65), (97,49), (129,37), (145,33), (193,25), (257,19), (289,17), (385,13), (513,10), (577,9), (769,7), (1153,5), (1537,4), (2305,3), (2,4609)		
10239	(2,10241), (3,5121), (5,2561), (6,2049), (9,1281), (11,1025), (17,641), (21,513), (33,321), (41,257), (65,161), (81,129), (129,81), (161,65), (257,41), (321,33), (513,21), (641,17), (1025,11), (1281,9), (2049,6), (2561,5), (5121,3), (10241,2)	12	12
22527	(2,22529),(3,11265),(5,5633),(9,2817), (12,2049),(17,1409),(23,1025),(33,705), (45,513),(65,353),(89,257),(129,177), (177,129),(257,89),(353,65),(513,45), (705,33),(1025,23),(1409,17),(2049,12), (2817,9),(5633,5),(11265,3),(22529,2)	12	12
49151	(2,49153),(3,24577),(4,16385),(5,12289), (7,8193),(9,6145),(13,4097),(17,3073), (25,2049),(33,1537),(49,1025),(65,769), (97,513),(129,385),(193,257),(257,193) (385,129),(513,97),(769,65),(1025,49), (1537,33),(2049,25),(3073,17),(4097,13), (6145,9),(8193,7),(12289,5),(16385,4), (24577,3),(49153,2)	30	-
106495	(2,106497),(3,53249),(5,26625), (9,13313),(14,8193),(17,6657), (27,4097),(33,3329),(53,2049), (65,1665),(105,1025),(129,833), (209,513),(257,417),(417,257), (513,209),(833,129),(1025,105), (1665,65),(2049,53),(3329,33),(4097,27), (6657,17),(8193,14),(13313,9),(26625,5), (53249,3),(106497,2)	20	8

In a similar manner, Tables 2.2, 2.3 exhibit Cullen Numbers, Motzkin Numbers, along with their corresponding rectangles satisfying (II.1).

Table No. 2.2: $A - S = \beta$

Cullen number (β)	$R(x, y)$	Observations	
		Primitive rectangles	Non- Primitive rectangles
1	(2,3),(3,2)	2	-
3	(2,5),(5,2)	2	-
9	(2,11),(3,6),(6,3),(11,2)	2	2
25	(2,27),(3,14), (14,3),(27,2)	4	-
65	(2,67),(3,34), (4,23), (7,12), (12,7),(23,4), (34,3),(67,2)	8	-
161	(2,163),(3,82),(4,55),(7,28), (10,19),(19,10),(28,7),(55,4), (82,3),(163,2)	8	2
385	(2,387),(3,194),(194,3),(387,2)	4	-
897	(2,899),(3,450),(450,3),(899,2)	2	2
2049	(2,2051),(3,1026),(6,411),(11,206), (26,83),(42,51),(51,42),(83,26), (206,11),(411,6),(1026,3),(2051,2)	6	6
4609	(2,4611),(3,2306),(6,923),(11,462), (462,11), (923,6),(2306,3),(4611,2)	6	2
10241	(2,10243),(3,5122),(4,3415),(7,1708), (10,1139),(19,570),(570,19),(1139,10), (1708,7),(3415,4),(5122,3),(10243,2)	8	4
22529	(2,22531),(3,11266),(4,7511),(6,4507), (7,3756),(11,2254),(16,1503),(31,752), (752,31),(1503,16),(2254,11),(3756,7), (4507,6),(7511,4),(11266,3),(22531,2)	16	-
49153	(2,49155),(3,24578),(8,7023),(15,3512), (3512,15),(7023,8),(24578,3),(49155,2)	8	-
106497	(2,106499),(3,53250),(8,15215), (15,7608),(7608,15),(15215,8), (53250,3),(106499,2)	4	4

Table No. 2.3: $A - S = \gamma$

Motzkin number (γ)	$R(x, y)$	Observations	
		Primitive rectangles	Non- Primitive rectangles
1	(2,3), (3,2)	2	-
2	(2,4),(4,2)	-	2
4	(2,6),(6,2)	-	2
9	(2,11),(3,6),(6,3),(11,2)	2	2
21	(2,23),(3,12), (12,3),(23,2)	2	2
51	(2,53),(3,27), (5,14), (14,5), (27,3),(53,2)	4	2
127	(2,129),(3,65),(5,33),(9,17), (17,9),(33,5),(65,3),(129,2)	8	-
323	(2,325),(3,163),(4,109),(5,82), (7,55),(10,37),(13,28),(28,13), (37,10),(55,7),(82,5),(109,4), (163,3),(325,2)	14	-
835	2,837),(3,419),(5,210),(12,77), (20,45),(23,39),(39,23),(45,20), (77,12),(210,5),(419,3),(837,2)	8	4
2188	(2,2190),(12,200),(200,12),(2190,2)	-	4
5798	(2,5800),(4,1934),(1934,4),(5800,2)	-	4
15511	(2,15513),(3,7757),(5,3879),(8,2217), (9,1940),(15,1109),(29,555),(57,278), (278,57),(555,29),(1109,15),(1940,9), (2217,8),(3879,5),(7757,3),(15513,2)	16	-
41835	(2,41837),(3,20919),(5,10460), (10460,5),(20919,3),(2,41837)	2	4
113634	(2,113636),(6,22728),(22728,6), (113636,2)	-	4

CONCLUSION

In this paper, we have presented rectangles such that, in each rectangle, the area added with its semi-perimeter as well as the area minus the semi-perimeter is represented by fascinating numbers, namely, Woodall Numbers, Cullen Numbers, Motzkin Numbers.

To conclude, one may search for rectangles with other characterization in connection with higher order fascinating numbers.

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