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Construction of Two Special Dio-Quadruples with Property D(2)

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ABSTRACT

This paper has two sections A and B. Section A deals with the study of formulating special dio-quadruples (a,b,c,d) such that the product of any two members of the set added with their sum and increased by two is a perfect square. Section B concerns with the construction of special dio-quadruples (a,b,c,d) such that the product of any two members of the set subtracted with their sum and increased by 2 is a perfect square.

INTRODUCTION

The problem of constructing the sets with property that the product of any two of its distinct elements is one less than a square has a very long history and such sets were studied by diophantus [1].A set of m positive integers $\{a_1, a_2, ..., a_m\}$ is said to have the property $D(n), n \in z - \{0\}$ if $a_i a_j + n$, a perfect square for all $1 \le i \le j \le m$ and such a set is called a diophantine m-tuples with property $D(n)$.Many mathematicians considered the construction of different formulations of Diophantine quadruples with the property $D(n)$ for any arbitrary integer n and also for any linear polynomials in n. This paper aims at constructing special dio-quadruples where the special mention is provided because it differs from the earlier one and the special dio-quadruples is constructed where the product of any two members of the quadruple with the addition or subtraction of members and increased by a non-zero integer or a polynomial with integer coefficients satisfies the required property [2-7]. This paper has two sections A and B. Section A deals with the study of formulating special dio-quadruples (a, b, c, d) such that the product of any two members of the set added with their sum and increased by two is a perfect square. Section B concerns with the construction of special dio-quadruples (a, b, c, d) such that the product of any two members of the set subtracted with their sum and increased by 2 is a perfect square.

METHOD OF ANALYSIS

Section A:

Let $a = 4n^2 - n - 1$ and $b = 15$ be two polynomials such that $ab + (a + b) + 2$ is a perfect square.

Let c_s be any non-zero polynomial such that

$$(a+1)c_s + a + 2 = p_s^2 \tag{1}$$

$$(b+1)c_s + b + 2 = q_s^2 \tag{2}$$

Eliminating c_s from (1) and (2), we get

$$(b+1)p_s^2 - (a+1)q_s^2 = (b - a) \tag{3}$$

Setting ,

$$p_s = X_s + (a+1)\,T_s \tag{4}$$

$$q_s = X_s + (b+1)\,T_s \tag{5}$$

in (3), we have

$$X_s^2 = (b+1)(a+1)\,T_s^2 + 1 \tag{6}$$

whose initial solution is $T_0 = 1, X_0 = 8n - 1$

The general solution of (6) is

$$X_s = \frac{1}{2} f_s\,,\; T_s = \frac{1}{2\sqrt{(a+1)(b+1)}} g_s \tag{7}$$

where

$$f_s = \left(X_0 + \sqrt{(a+1)(b+1)}T_0\right)^{s+1} + \left(X_0 - \sqrt{(a+1)(b+1)}T_0\right)^{s+1}$$
$$g_s = \left(X_0 + \sqrt{(a+1)(b+1)}T_0\right)^{s+1} - \left(X_0 - \sqrt{(a+1)(b+1)}T_0\right)^{s+1}$$

Substitution of (7) in (4) gives

$$p_s = \frac{1}{2} f_s + (a+1) * \frac{1}{2\sqrt{(a+1)(b+1)}} g_s \tag{8}$$

From (8) and (1) we get

$$c_s = \frac{(p_s^2 - 1)}{(a+1)} - 1 \tag{9}$$

Substituting $s = 0$ in (9) we have,

$$c_0 = \frac{(p_0^2 - 1)}{(a+1)} - 1$$

(i.e), $c_0 = 4n^2 + 15n + 13$

Note that, the tuple (a, b, c_0) is a dio-triple with property $D(2)$.

Again, substituting $s = 1$ in (9) and simplifying we get,

$$c_1 = 1024n^4 + 3584n^3 + 2640n^2 - 868n + 59$$

It is seen that $(a, b, c_0(x), c_1(x))$ represent Dio-quadruples with property $D(2)$respectively.

In general, it is observed that the quadruple $(a, b, c_{s-1}(x), c_s(x)), s = 1,2,3,........$ is a dio-quadruple with property $D(2)$. Some numerical examples are given in Table 1 below:

Table No. 1: $D(2)$Dio-quadruples

n	(a,b,c_0,c_1)	(a,b,c_1,c_2)	(a,b,c_2,c_3)
1	(2,15,32,6439)	(2,15,6439,1249364)	(2,15,1249364,242370407)
2	(13,15,59,53939)	(13,15,53939,48438119)	(13,15, 48438119,43497377879)

Section B:

Let $a = 3n^2 - n + 1$ and $b = 13$ be two polynomials such that $ab - (a + b) + 2$ is a perfect square.

Let c_s be any non-zero polynomial such that

$$(a - 1)c_s - a + 2 = p_s^2 \tag{10}$$

$$(b - 1)c_s - b + 2 = q_s^2 \tag{11}$$

Eliminating c_s from (10) and (11), we get

$$(b - 1)p_s^2 - (a - 1)q_s^2 = (b - a) \tag{12}$$

Setting

$$p_s = X_s + (a - 1)\, T_s \tag{13}$$

$$q_s = X_s + (b-1)T_s \tag{14}$$

in (12), we have

$$X_s^2 = (b-1)(a-1)T_s^2 + 1 \tag{15}$$

whose initial solution is $T_0 = 1, X_0 = 6n-1$

The general solution of (15) is

$$X_s = \frac{1}{2}f_s,\ T_s = \frac{1}{2\sqrt{(a-1)(b-1)}}g_s \tag{16}$$

where

$$f_s = \left(X_0 + \sqrt{(a-1)(b-1)}T_0\right)^{s+1} + \left(X_0 - \sqrt{(a-1)(b-1)}T_0\right)^{s+1}$$
$$g_s = \left(X_0 + \sqrt{(a-1)(b-1)}T_0\right)^{s+1} - \left(X_0 - \sqrt{(a-1)(b-1)}T_0\right)^{s+1}$$

Substitution of (16) in (13) gives

$$p_s = \frac{1}{2}f_s + (a-1)*\frac{1}{2\sqrt{(a-1)(b-1)}}g_s \tag{17}$$

From (17) and (10) we get

$$c_s = \frac{(p_s^2-1)}{(a-1)} + 1 \tag{18}$$

Substituting $s = 0$ in (18) we have,

$$c_0 = \frac{(p_0^2-1)}{(a-1)} + 1$$

(i.e), $c_0 = 3n^2 + 11n + 11$

Note that, the tuple (a, b, c_0) is a dio-triple with property $D(2)$.

Again, substituting $s = 1$ in (18) and simplifying we get,

$$c_1 = 432n^4 + 1440n^3 + 924n^2 - 460n + 45$$

It is seen that $(a, b, c_0(x), c_1(x))$ represent Dio-quadruples with property $D(2)$respectively.

In general, it is observed that the quadruple $(a, b, c_{s-1}(x), c_s(x)), s = 1,2,3,........$ is a dio-quadruple with property $D(2)$. Some numerical examples are given in Table 2 below:

Table No. 2: $D(2)$Dio-quadruples

n	(a,b,c_0,c_1)	(a,b,c_1,c_2)	(a,b,c_2,c_3)
1	(3,13,25,1405)	(3,13,1405,2381)	(3,13,2381,22855561)
2	(11,13,45,21253)	(11,13,21253,10243465)	(11,13, 10243465 ,4937328439)

CONCLUSION

In this paper, we have illustrated the process of obtaining special Dio-quadruples with property $D(2)$. To conclude, one may search for special Dio-quadruples with suitable properties.

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