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Estimation of Exponential-Logarithmic Distribution Based on the Doubly Censored Data with Lifetime Application

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R. A. Bakoban

Department of Statistics, Faculty of Sciences, King Abdulaziz University, Jeddah, Saudi Arabia.

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ABSTRACT

In this paper, the exponential-logarithmic distribution (ELD) is considered. Classical and Bayesian analysis using Markov Chain Monte Carlo (MCMC) method are studied. It is assumed that the lifetimes of test units follow ELD. Based on type II doubly censoring, the maximum likelihood estimations (MLEs) are obtained for the distribution parameters. Also, asymptotic variance and covariance matrix of the estimators are investigated. An iterative procedure is used to obtain the estimators numerically using Mathematica 9. In addition, confidence intervals of the estimators are presented. Different loss functions are used to discuss Bayesian estimation. A real data set is analyzed to illustrate the proposed methods and compare the performance of the estimates.

INTRODUCTION

The exponential-logarithmic distribution (ELD) was introduced by Tahmasbi and Rezaei (2008). They constructed this model as a log-series mixture of exponential random variables. ELD has decreasing failure rate thus making it a suitable model for studying lengths of organisms, devices, and materials in biological and engineering fields. Moala and Garcia (2013) pointed that ELD could be a good alternative to analyze lifetime data because the survival and hazard functions presented a closed analytical form. This would imply analytical flexibility and computational advantages for inference analysis. Moala and Garcia (2013) considered the estimation of ELD using maximum likelihood (ML) and Bayesian methods. They studied uniform, beta and Jeffreys priors for Bayesian analysis using Markov Chain Monte Carlo (MCMC) method based on complete samples. They found that there is no difference between the priors when the sample size is moderately large. Although, in small dataset, the uniform prior is more suitable for the estimation of parameter *p*. Pappas et al. (2015) presented a generalization of ELD, which has increasing, decreasing and unimodal failure rates. They estimated the parameters using maximum likelihood estimation based on a complete method and when some observations are randomly right-censored. Also, Pijyan (2015) used pseudo maximum likelihood method to estimate the parameters' distribution based on censored data.

The probability density function (PDF) and the cumulative distribution function (CDF) are given, respectively, as:

$$f(x;p,\beta) = -\left(\frac{1}{\log p}\right)\frac{\beta(1-p)e^{-\beta x}}{1-(1-p)e^{-\beta x}},\ x>0 \tag{1}$$

$$F(x;p,\beta) = 1-\frac{\log[1-(1-p)e^{-\beta x}]}{\log p} \tag{2}$$

Where, $0<p<1$ and $\beta>0$ are the parameters. It is worth noting that when $p\to 1$, the ELD reduces to the exponential distribution with parameter β. Tahmasbi and Rezaei (2008) studied several statistical properties of ELD.

In this study, we considered the type II doubly censored samples denoted by:

$$x_r \le x_{r+1} \le \ldots \le x_{s-1} \le x_s,\ 1\le r\le s\le n.$$

The samples are obtained when some observations are initially censored and where the life test is terminated before all items on test failed. Type II doubly censored samples have been investigated by several authors among them; Fernández (2000 a, b), Raqab and Madi (2002), Kim and Song (2010), Khan (2014) and Shen et al. (2016).

The main objective of this paper was to estimate the distributions' parameters using maximum likelihood and Bayesian methods. In Section 2, MLEs are derived. For Bayesian inference discussed in Section 3, MCMC are used for computing Bayesian estimates. Three different kinds of loss functions are investigated. In Section 4, an application on the real dataset carried for comparisons between estimators are discussed. Finally, in Section 5, we made some conclusions.

2. Maximum Likelihood Estimation

Consider a random sample of size n from ELD and let $x_r \leq \ldots \leq x_s$ be the ordered observations remaining when the $(r-1) = nq_1$ smallest observations and $(n-s) = nq_2$ largest observations have been censored; where, $r < s, q_1$ and q_2 are fixed, and $0 \leq q_1 + q_2 < 1.$ The likelihood function, given the type II doubly censored sample $\underline{x} = (x_r, \ldots, x_s),$ can be written as:

$$L(p,\beta|\underline{x}) = \frac{n!}{(r-1)!(n-s)!}\prod_{i=r}^{s} f\,(x_i;p,\beta)\{F(x_r;p,\beta)\}^{r-1}\{1-F(x_s;p,\beta)\}^{n-s}. \qquad (3)$$

Putting Equations (1) and (2) in (3), we get the likelihood function of ELD as follows:

$$L(p,\beta|\underline{x}) = \frac{n!}{(r-1)!(n-s)!}\{1-\frac{\log[1-(1-p)e^{-\beta x_r}]}{\log p}\}^{r-1}\{\frac{\log[1-(1-p)e^{-\beta x_s}]}{\log p}\}^{n-s}.$$

$$\times\prod_{i=r}^{s}\frac{-\beta(1-p)e^{-\beta x_i}]}{\log p\,[1-(1-p)e^{-\beta x_i}]}. \qquad (4)$$

Accordingly, the log-likelihood of Equation (4) is given by:

$$\ell = \log L \propto (r-1)\log[1-\frac{\log\phi_r}{\log p}] + (n-s)\log[\frac{\log\phi_s}{\log p}]$$

$$+\sum_{i=r}^{s}[\log(\frac{-1}{\log p}) + \log\beta + \log(1-p) - \beta x_i - \log\phi_i]$$

Where, $\phi_i = \phi(\beta, x_i) = 1-(1-p)e^{-\beta x_i}$.

Thus, the maximum likelihood estimators (MLEs) of the parameters p and β could be found by solving the two non-linear equations simultaneously.

$$\frac{\partial \ell}{\partial p} = \frac{\partial \log L}{\partial p} = -(r-1)(1-\frac{\log \phi_r}{\log p})^{-1}(\log p)^{-2}[\phi_r^{-1}e^{-\beta x_r}\log p - p^{-1}\log \phi_r]$$

$$+(n-s)[\phi_s^{-1}(\log \phi_s)^{-1}e^{-\beta x_s} - (p\log p)^{-1}]$$

$$-(s-r+1)[(p\log p)^{-1}+(1-p)^{-1}] - \sum_{i=r}^{s}\phi_i^{-1}e^{-\beta x_i} \quad (5)$$

$$\frac{\partial \log L}{\partial \beta} = (r-1)(1-\frac{\log \phi_r}{\log p})^{-1}(\frac{-1}{\log p})\phi_r^{-1}\phi_r'$$

$$+(n-s)(\log \phi_s)^{-1}\phi_s^{-1}\phi_s' + (s-r+1)\beta^{-1}$$

$$-\sum_{i=r}^{s} x_i - \sum_{i=r}^{s}\phi_i^{-1}\phi_i' \quad (6)$$

Where, $\phi_i' = \frac{\partial \phi_i}{\partial \beta}$.

The solution of Equations (5) and (6) can be found using Newton-Raphson method.

The asymptotic variance-covariance of the MLE for the parameters p and β are given by the elements of the inverse of the Fisher information matrix

$$\mathrm{I}_{ij} = -E\,(\frac{\partial^2 l}{\partial \varphi_i\, \partial \varphi_j}),\ \ i, j = 1, 2,\ \ \underline{\varphi} = (\beta, p). \quad (7)$$

Although it is difficult to get the exact expectation of the above expression, we will take the approximate asymptotic variance-covariance matrix; the observed variance-covariance, for MLE by dropping the expectation from Equation (7). Thus, the observed variance-covariance matrix is obtained by inverting observed information matrix as follows:

$$\mathrm{I}^{-1}(p,\beta) = \begin{bmatrix} \mathrm{var}(\hat{p}) & \mathrm{cov}(\hat{p},\hat{\beta}) \\ \mathrm{cov}(\hat{p},\hat{\beta}) & \mathrm{var}(\hat{\beta}) \end{bmatrix}. \quad (8)$$

The asymptotic normality of the MLE can be used to compute the approximate confidence intervals for the parameters p and β, which become:

$$(\hat{p} \pm z_{\alpha/2}\sqrt{\text{var}(\hat{p})}) \quad \text{and} \quad (\hat{\beta} \pm z_{\alpha/2}\sqrt{\text{var}(\hat{\beta})}) \tag{9}$$

Where, $z_{\alpha/2}$ is the upper $(\alpha/2)100pth$ percentile of the standard normal variate.

3. Bayesian Estimation

In this section, we present the different kinds of loss functions (see also: Soliman et al. (2012), Feroze and Aslam (2012), Feroze et al. (2014) and Bakoban and AbuBaker (2015) and Feroze (2016)) to estimate the parameters.

Under the assumption that both of the parameters p and β are unknown, it is assumed that the parameter β has a gamma prior and p has a uniform prior U(0,1). Thus, the joint prior density of the parameters p and β can be written as:

$$\pi(p,\beta) = \frac{b^a}{\Gamma(a)}\beta^{a-1}e^{-b\beta},\ 0 < p < 1,\ \beta > 0. \tag{10}$$

Where, a and b are the hyperparameters of gamma prior.

Using Bayesian theorem, by multiplying Equations (4) and (10), the joint posterior density function of p and β given the data could be written as:

$$\pi^*(p,\beta|\underline{x}) = \frac{L(\underline{x}|p,\beta)\pi(p,\beta)}{\int_0^\infty\int_0^\infty L(\underline{x}|p,\beta)\pi(p,\beta)dp\,d\beta} \tag{11}$$

Therefore, the Bayes estimate of any function of p and β say $\underline{\varphi} = (\beta, p)$, under squared error (SE) loss function is:

$$\hat{\varphi}_{BS}(p,\beta) = E_{p,\beta|\underline{x}}(\varphi(p,\beta))$$
$$= \frac{\int_0^\infty\int_0^\infty \varphi(p,\beta)L(\underline{x}|p,\beta)\pi(p,\beta)dp\,d\beta}{\int_0^\infty\int_0^\infty L(\underline{x}|p,\beta)\pi(p,\beta)dp\,d\beta}. \tag{12}$$

Another kind of loss functions is a LINEX (linear-exponential) loss function. The LINEX loss function gives more weight to overestimation or underestimation. The Bayes estimate of any function of p and β say $\underline{\varphi} = (\beta, p)$, under LINEX loss function is:

$$\hat{\varphi}_{BL}(p,\beta) = \frac{-1}{c}\log[E_{p,\beta|\underline{x}}(e^{-c\varphi(p,\beta)})],\ c \neq 0$$
$$= \frac{\int_0^\infty\int_0^\infty e^{-c\varphi(p,\beta)} L(\underline{x}|p,\beta)\pi(p,\beta)dp\,d\beta}{\int_0^\infty\int_0^\infty L(\underline{x}|p,\beta)\pi(p,\beta)dp\,d\beta}. \tag{13}$$

A more general loss function is the general entropy loss function (GE). The Bayes estimate of any function of p and β say $\underline{\varphi} = (\beta, p)$, under GE loss function is:

$$\hat{\varphi}_{BG}(p,\beta) = [E_{p,\beta|\underline{x}}(\varphi^{-q}(p,\beta))]^{-1/q}$$
$$= \frac{\int_0^\infty\int_0^\infty \varphi^{-q}(p,\beta) L(\underline{x}|p,\beta)\pi(p,\beta)dp\,d\beta}{\int_0^\infty\int_0^\infty L(\underline{x}|p,\beta)\pi(p,\beta)dp\,d\beta}. \tag{14}$$

Generally, the ratio of two integrals given by Equations (12-14) cannot be obtained in a closed form. Therefore, we use the MCMC method to generate samples from the posterior distributions and then compute the Bayes estimators of $\underline{\varphi} = (\beta, p)$ under the different kinds of loss functions. The MCMC approach is described as follows:

3.1 MCMC techniques

Monte Carlo integration is the main problem applied to obtain samples from very complex probability distribution. Markov Chain is a stochastic process in which future states are independent of past states given the present state. Markov chain Monte Carlo (MCMC) method use computer simulation of Markov chains in the parameter space (Gilks et al. (1996) and Gamerman (1997)). Therefore, MCMC is a class of methods for simulating draws that are slightly dependent and approximate from a (posterior) distribution. We then record those draws and calculate the quantities of interest for the (posterior) distribution. In Bayesian statistics, there are two MCMC algorithms commonly used namely, the Gibbs Sampler and the Metropolis-Hastings algorithm.

1. Gibbs Sampling

The Gibbs sampler can be used to sample from the joint distribution if the full conditional distributions for each parameter is known. For each parameter, the full conditional distribution is the distribution of the parameters on the known information and all other parameters. Suppose we have a posterior to sample from and the full conditional distributions remain unknown then there will be no Gibbs sampling. If this sampling method fails, then we can resort to the Metropolis-Hastings algorithm, which will always work.

2. Metropolis-Hastings Algorithm

The Metropolis-Hastings (MH) algorithm follows the following steps:

1. Choose a starting value $\theta^{(0)}$, such that the probability $p(\theta^{(0)}|y) > 0$.

2. At iteration *t*, draw a candidate θ^{*} from a jumping distribution $J_t(\theta^{*}|\theta^{(t-1)})$.

3. Compute an acceptance ratio (probability):

$$r = \frac{p(\theta^{*}|y)/J_t(\theta^{*}|\theta^{(t-1)})}{p(\theta^{(t-1)}|y)/J_t(\theta^{(t-1)}|\theta^{*})} \qquad (15)$$

4. Accept θ^{*} as $\theta^{(t)}$ with probability min(*r*, 1). If θ^{*} is not accepted, then $\theta^{(t)} = \theta^{(t-1)}$.

5. Repeat steps 2-4 *N* times to get *N* draws from $p(\theta|y)$, with optional burn-in.

The original Metropolis algorithm required that $J_t(\theta^{*}|\theta^{(t-1)})$ be a symmetric distribution (such as the normal distribution), that is $J_t(\theta^{*}|\theta^{(t-1)}) = J_t(\theta^{(t-1)}|\theta^{*})$ using the Metropolis-Hastings algorithm, it is evident that symmetry is unnecessary.

If we have a symmetric jumping distribution that is dependent on $\theta^{(t-1)}$, then we have what is known as random walk Metropolis sampling.

In the case where our jumping distribution is symmetric,

$$r = \frac{p(\theta^{*}|y)}{p(\theta^{(t-1)}|y)}$$

If our candidate draw has higher probability than our current draw, then our candidate is better hence, we definitely accept.

In the case where our jumping distribution is not symmetric, we use Equation (15).

In the case of independent Metropolis-Hastings sampling,

$$r = \frac{p(\theta^* | y) / J_t(\theta^*)}{p(\theta^{(t-1)} | y) / J_t(\theta^{(t-1)})}, \quad \text{where } J_t(\theta^* | \theta^{(t-1)}) = J_t(\theta^*).$$

Accept θ^* as $\theta^{(t)}$ with probability min(*r*, 1). If θ^* is not accepted, then $\theta^{(t)} = \theta^{(t-1)}$.

1. For each θ^*, draw a value *u* from the Uniform (0,1) distribution.

2. If $u \le r$,, accept θ^* as $\theta^{(t)}$. Otherwise, use $\theta^{(t-1)}$ as $\theta^{(t)}$

Candidate draws with higher density than the current draw is always accepted.
Unlike in rejection sampling, each iteration always produces a draw, either θ^* or $\theta^{(t-1)}$.

3.2 Bayesian estimation using MCMC techniques

In this subsection, Bayes estimators are derived using MCMC. Moala and Garcia (2013) concluded that the uniform distribution is a suitable prior for the parameter *p* in the case of small samples. It is better for estimating the parameters than beta and Jefferys priors. In conclusion, a Uniform prior U(0, 1) for the parameter *p* is used in this sub-section.

The joint posterior density function of p and β presented, in Equation (11), could be written as follows:

$$\pi^*(p, \beta | \underline{x}) \propto \beta^{a+s-r} e^{-b\beta+v} \qquad (16)$$

Where, $v = (r-1)\log(1 - \frac{\log \phi_r}{\log p}) + (n-s)[\log(\log \phi_s) - \log(\log p)]$

$$+ \sum_{i=r}^{s} \{\log(\frac{-1}{\log p}) + \log(1-p) - \log(\phi_i) - \beta x_i\}.$$

The conditional pdfs of *p* and β are given by

$$\pi_1^*(\beta | p, \underline{x}) \sim Gamma(a+s-r+1, b+\sum_{i=r}^{s} x_i) \qquad (17)$$

$$\text{and } \pi_2^*(p\,|\,\beta, \underline{x}) \propto e^{v+\beta\sum_{i=r}^{s} x_i}. \qquad (18)$$

In this representation, the full conditional form presented in Equation (17) is the gamma density with shape parameter $(a+s-r+1)$ and scale parameter $(b+\sum_{i=r}^{s} x_i)$. Thus, the samples of β can be easily generated using any gamma generating routine. Also, since the conditional posterior of p in Equation (18) do not present standard forms, and therefore Gibbs sampling is not a straightforward option; the use of the Metropolis–Hasting sampler is required for the implementations of MCMC methodology. Given these conditional distributions in Equations (17) and (18), Metropolis-within-Gibbs samplers are used.

The following algorithm is a hybrid algorithm with Gibbs sampling steps for drawing the parameter β and with MH steps for drawing p:

1. Start with initial value, say, MLE ($p_{MLE}^{(0)}$, $\beta_{MLE}^{(0)}$).

2. Set j = 1.

3. Generate $\beta^{(j)}$ from Gamma $(a+s-r+1, b+\sum_{i=r}^{s} x_i)$.

4. Using the following Metropolis–Hastings, generate $p^{(j)}$ from the proposal distribution.

i) Generate a proposal p^* from the proposal distribution (the Normal distribution $N(p^{(j-1)}, var(\hat{p}))$ is used).

ii) Evaluate the acceptance probability

$$\rho = \min\left[1, \frac{\pi_2^*(p^*\,|\,\underline{x}, \beta^{(j)})}{\pi_2^*(p^{(j-1)}\,|\,\underline{x}, \beta^{(j)})}\right].$$

iii) Generate u from a Uniform (0, 1) distribution.

iv) If $u \le \rho$, accept the proposal and set $p^{(j)} = p^*$, else set $p^{(j)} = p^{(j-1)}$.

5. Set j = j +1.

6. Repeat steps 3–5, N times and obtain $\beta^{(j)}$and $p^{(j)}$, $i = 1,...,N$.

7. To compute the credible intervals of β *and* p, order $\beta^{(j)}$ and $p^{(j)}$, $i = 1,...,N$, as

$$\beta_{(1)} < ... < \beta_{(N)} \text{ and } p_{(1)} < ... < p_{(N)}.$$

Then, the $100(1-\gamma)\%$ credible intervals of $\underline{\varphi} = (\beta, p,)$ become $(\varphi_{(N\gamma/2)}, \varphi_{(N(1-\gamma/2))})$.

In order to guarantee the convergence and remove the affection of the selection of initial values, the first M simulated varieties are discarded. Then the selected samples are $\beta^{(j)}$ and $p^{(j)}$, $i = M+1,...,N$, for sufficiently large N, forms approximate the posterior samples used to develop the Bayesian inferences.

Based on SE, presented in Equation (12), the approximate Bayesian estimates of $\underline{\varphi} = (\beta, p)$ is given as:

$$\hat{\varphi}_{BS} = \frac{1}{N-M}\sum_{j=M+1}^{N} \varphi_i^{(j)}$$

Also, the approximate Bayesian estimates for $\underline{\varphi}$, under LINEX loss function, from Equation (13) is given as:

$$\hat{\varphi}_{BL} = \frac{-1}{c}\log\left[\frac{1}{N-M}\sum_{j=M+1}^{N} e^{-c\,\varphi_i^{(j)}}\right],$$

And the approximate Bayes estimates for $\underline{\varphi}$, based on the GE, from Equation (14) is given as:

$$\hat{\varphi}_{BG} = \left[\frac{1}{N-M}\sum_{j=M+1}^{N} (\varphi_i^{(j)})^{-q}\right]^{-1/q}.$$

4. Real Data Application

In this section, a real data set is analyzed to illustrate the methods of estimations discussed in the previous sections. Lawless (1982) considered dataset related to the lifetime of a type of electrical insulator subject to a constant voltage stress. The dataset (lifetimes in minutes to fail) is as follows:

0.96, 4.15, 0.19, 0.78, 8.01, 31.75, 7.35, 6.50, 8.27, 33.91, 32.52, 3.16, 4.85, 2.78, 4.67, 1.31, 12.06, 36.71, 72.89.

Moala and Garcia (2013) proved that ELD is appropriate for the data and gives better fit than Weibull distribution proposed by Lawless (1982) to fit the data. Therefore, Lawless dataset is used to study the performance of the non-Bayesian and Bayesian estimators which were derived in Sections 3 and 4. Complete and doubly censored samples with $(r,s)=(1,19)$ and $(5,15)$ were used. Based on doubly censored samples, the MLE and 95% C.I. of the parameters β and p are presented in Tables 1, 2 and 3. For Bayesian estimators, MCMC method is conducted using the algorithms in Sub-Section 3.2. Based on doubly censored samples of the Bayesian estimate under SE, LINEX and GE loss functions of the parameters β and p are presented in Tables 1, 2 and 3. For the LINEX and GE loss functions, $c=3,-3$ and $q=3,-3$ are used, respectively.

MH subclass of MCMC was carried with N=11000 and M=1000. The hyperparameters of gamma prior $a=10$ and $b=500$ were used to generate the values of β.

Table 1. MLE and Bayes MCMC estimates under SE, LINEX and GE of β for Lawless data.

r	s	MLE	SE	LINEX		GE	
				$c=-3$	$c=3$	$q=-3$	$q=3$
1	19	0.03934	0.0374	0.03745	0.03731	0.03860	0.0348
5	15	0.02403	0.0353	0.03538	0.03521	0.0369	0.0319

Table 2. MLE and Bayes MCMC estimates under SE, LINEX and GE of p for Lawless data.

r	s	MLE	SE	LINEX		GE	
				$c=-3$	$c=3$	$q=-3$	$q=3$
1	19	0.09817	0.2124	0.25648	0.1846	0.3088	0.0524
5	15	0.04644	0.2401	0.30815	0.20096	0.3630	0.0623

Tables 1 and 2 show that means for estimators of β and p based on censored samples were less than the others based on complete sample. Also, the Bayes estimate under GE (with q =3) was the best for complete sample whereas; the MLEs seem to be the best for censored sample. Moreover, according to the 95% CIs, Table 3 shows that the length of intervals for the estimate of β based on MCMC method are less than the others based on ML method in the both cases (complete and censored samples). It is also the same for the estimate of p in the case of complete sample. Further, Intervals based on MLEs in the case of doubly censored sample was shorter than the other of complete sample.

Table 3. 95% CIs MLE and Bayes MCMC estimates under SE of β and p for Lawless data.

r	s	Method	β		p	
			C.I.	Length	C.I.	Length
1	19	MLE	(0.00271, 0.07596)	0.07324	(-0.14277, 0.33911)	0.48189
		MCMC	(0.0268, 0.0494)	0.02261	(0.0445, 0.5238)	0.47931
5	15	MLE	(-0.01147, 0.05955)	0.07103	(-0.09697, 0.18985)	0.28682
		MCMC	(0.0236, 0.0489)	0.02527	(0.048, 0.6367)	0.58871

Figures 1, 2, 5 and 6 show the values of the parameter β and p obtained from MCMC for complete sample and doubly censored samples. Figures 3, 4, 7 and 8, also show the histograms of the parameter β and p obtained from MCMC for complete sample and doubly censored samples. It is noted that the histograms of the parameter β are approximately, normally distributed, while the histograms of the parameter p are right skewed. Table 4 shows some statistical measures, mean, median, standard deviation (S.D.) and skewness (S.K.), after bun in; for *N-M*, for posterior distributions of β and p based on MCMC.

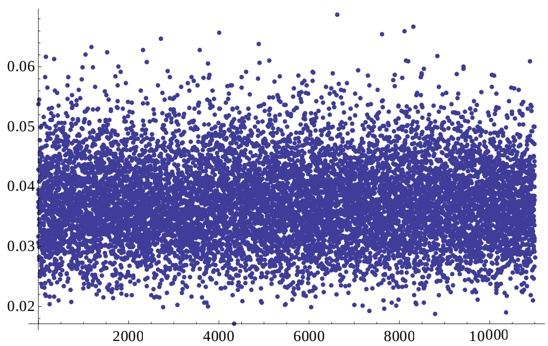

Figure 1. Values of the parameter β obtained from MCMC for complete sample.

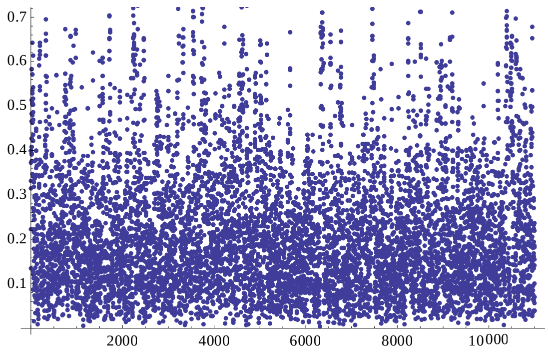

Figure 2. Values of the parameter p obtained from MCMC for complete sample.

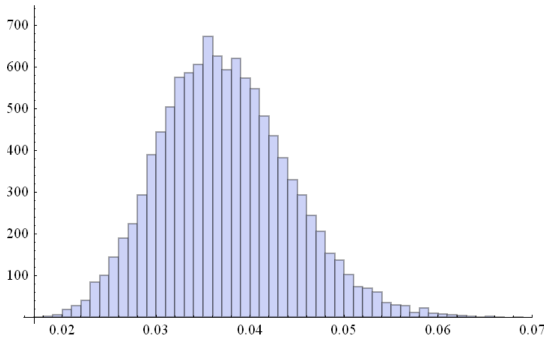

Figure 3. Histogram of the parameter β obtained from MCMC for complete sample.

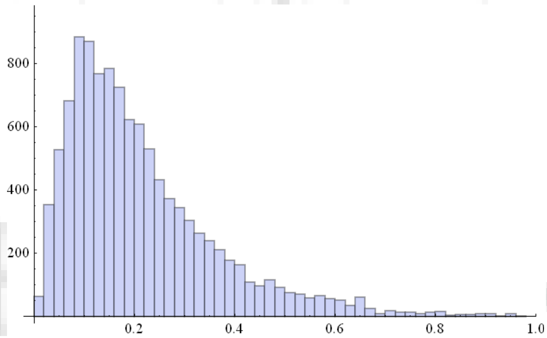

Figure 4. Histogram of the parameter p obtained from MCMC for complete sample.

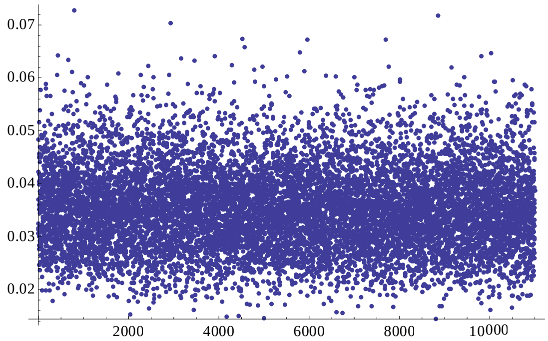

Figure 5. Values of the parameter β obtained from MCMC for doubly censored sample.

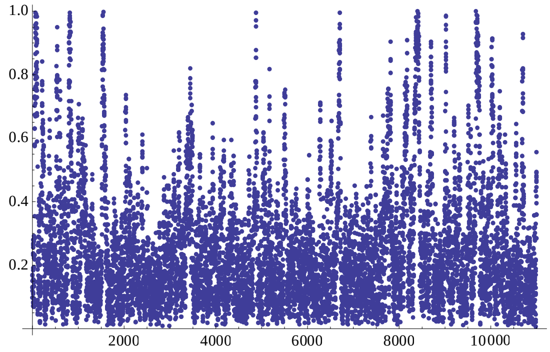

Figure 6. Values of the parameter p obtained from MCMC for doubly censored sample.

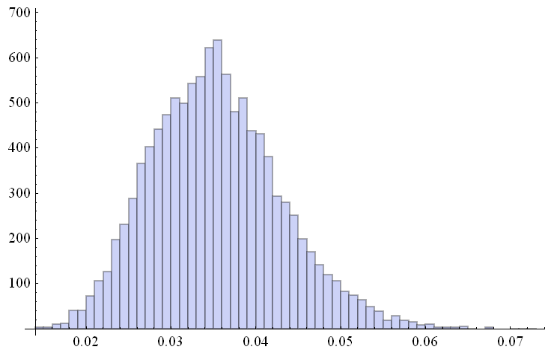

Figure 7. Histogram of the parameter β obtained from MCMC for doubly censored sample.

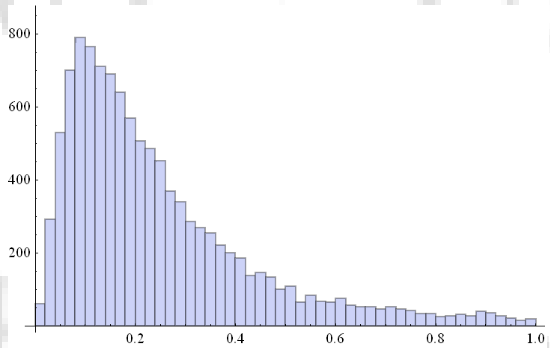

Figure 8. Histogram of the parameter p obtained from MCMC for doubly censored sample.

Table 4. MCMC results for some posterior statistical measures for Lawless data.

Parameter	mean	median	S.D.	S.K.
Complete sample r =1, s =19				
β	0.03738	0.03693	0.00689	0.36857
p	0.21237	0.17367	0.15112	1.53756
Doubly censored sample r =5, s =15				
β	0.03530	0.03485	0.00767	0.42313
p	0.24012	0.18851	0.18389	1.54736

Finally, some of the computing criteria (Linhart and Zucchini, 1986) are presented in Table 5. AIC (Akaike information criterion), BIC (Bayesian information criterion), CAIC (Consistent Akaike information criterion), HQIC (Hannan- Quinn information criterion), K-S (Kolmogorov- Smirnov) and the value -2 l, $l = \log L$ are computed for all estimators presented in Tables 1 and 2. In Table 5, in the case of complete sample, the estimated ELD provide excellent good fit to the given data and the MLEs estimates fits the data better than Bayes based on all computing criteria except for K-S, where the Bayes estimate under general entropy with q =3 was fitting better. On the other hand, in the case of doubly censored sample, MLEs and the Bayes estimate under general entropy with q =3 were fitting better and close to each other than the overall estimates. Furthermore, all computing criteria for doubly censored sample were less than the corresponding complete sample. This implies that the censored sample is more useful especially in real-life applications.

Table 5. Computing criteria for ML and Bayes estimates.

Type of sample	Methods of estimation	AIC	BIC	CAIC	HQIC	K-S	-2 l
Complete sample	MLE	139.983	141.872	140.733	140.303	0.1382	135.983
	SE	141.238	143.126	141.988	141.557	0.24118	137.238
	LINEX ($c=-3$)	141.828	143.717	142.578	142.147	0.26366	137.828
	GE ($q=-3$)	142.269	144.158	143.019	142.588	0.27802	138.269
	LINEX ($c=3$)	140.879	142.768	141.629	141.199	0.22490	136.879
	GE ($q=3$)	140.270	142.159	141.020	140.59	0.13540	136.27
Doubly censored sample	MLE	103.048	103.844	104.548	102.547	0.52290	99.048
	SE	104.77	105.565	106.27	104.268	0.61854	100.77
	LINEX ($c=-3$)	105.629	106.425	107.129	105.128	0.64754	101.629
	GE ($q=-3$)	106.015	106.811	107.515	105.514	0.65615	102.015
	LINEX ($c=3$)	104.28	105.076	105.78	103.778	0.59782	100.28
	GE ($q=3$)	103.288	104.084	104.788	102.787	0.48767	99.288

5. CONCLUSION

We considered using the ELD for the purpose of this study. Point and interval estimation were derived. ML and Bayesian estimations were used. MCMC technique was conducted for Bayesian estimates under SE, LINEX and GE loss functions. The study proposed the use of gamma and Uniform (0, 1) priors for Bayes estimation. The method applied was discussed by Lawless (1982) and Moala and Garcia (2013). Based on complete and doubly censored samples, we found that most Bayes estimators behave better than others, especially under GE loss function. This point agrees with the conclusion of Moala and Garcia (2013) on complete sample. Finally, the doubly censored sample was fitting better based on the computing criteria.

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